Monopole Condensation and Color Confinement

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New evidence is discussed of monopole condensation in the vacuum of $SU(2)$ and $SU(3)$ gauge theories. Monopoles defined by different abelian projections do condense in the transition to the confined phase and show the same behavior. For $SU(2)$ critical indices are determined by finite size scaling analysis and the results agree with the 3d Ising Model, as expected.

1. INTRODUCTION

A particularly interesting attempt to understand the structure of the QCD vacuum and color confinement is the dual superconductor mechanism [1,2]. By dual Meissner effect, in the presence of a quark-antiquark pair the chromoelectric field is squeezed into flux tubes and the potential energy rises linearly with the distance producing confinement.

The detection on the lattice of linear rising potential [3] and of flux tube configurations [4] has given support to this idea. Direct evidence may come from the study of the symmetry of QCD vacuum. Dual superconductivity is indeed the spontaneous breaking of the $U(1)$ symmetry related to monopole charge conservation. The color deconfining transition occurs between the broken magnetic phase, where magnetic monopoles condense, and the symmetric phase where the vacuum is invariant under the action of U(1) magnetic group.

This order-disorder transition can be studied by constructing an operator with non trivial magnetic charge and computing its vacuum expectation value (*v.e.v.*) [5]. A creation operator for a monopole can be defined and its *v.e.v.* used as an order parameter.

In non abelian gauge theories monopoles can be defined by means of abelian projection [6] in analogy with the 't Hooft-Polyakov monopole [7,8] in the Georgi-Glashow model. In case of $SU(2)$ or $SU(3)$ pure gauge theories, as well as QCD, there is no fundamental Higgs fields, but any operator $\Phi(x)$ in the adjoint representation of the gauge group can play its role.

The abelian projection is set diagonalizing the Φ operator. Under the residual $U(1)^{N-1}$ gauge subgroup the diagonal components of the vector field transform as abelian fields. There are abelian monopoles in the sites where $\Phi(x)$ has two equal eigenvalues and the diagonalization is singular. In $SU(2)$ gauge theories this means that $\vec{\Phi}(x)\cdot\sigma$ is zero. It is worth underlining that monopole charge is color singlet and gauge invariant. Condensation is no breaking of the gauge symmetry group.

A priori on the lattice parallel transport on any closed path can be chosen as $\Phi(x)$. There are an infinite number of possible choices, for example the open plaquette $U_{XY}(x)$, the open Polyakov loop $P(x)$ and the "butterfly" $U_{XY}U_{ZT}$. The question is: What kind of monopoles do condense if any? Even if on a single configuration the number and the location of monopoles depend on the choice of the abelian projection, 't Hooft [6] guessed that different monopoles are physically equivalent on the average and do condense in the QCD vacuum.

The creation operator of a monopole $\mu(\vec{y},t)$ shifts field configurations $|a_\mu(\vec{x},t)\rangle$ by a field $a_\mu^{(top)}(\vec{x}-\vec{y},t)$ with non trivial magnetic charge (for example the Dirac potential)

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$$\mu(\vec{y},t)|a_\mu(\vec{x},t)\rangle = |a_\mu(\vec{x},t) + a_\mu^{(top)}(\vec{x}-\vec{y},t)\rangle \ . \quad (1)$$

The original construction goes back to [9] and has been developed in different forms and different models by many authors [5,10–12]. Dealing with compact gauge groups technical modification are needed due to the compact nature of field variables that cannot be shifted arbitrary [5]. For details about non abelian gauge theories [13,14]. The $v.e.v$ of μ is given by

$$\langle\mu(\vec{y},t)\rangle = \frac{Z[S_m]}{Z[S]} \quad (2)$$

where S is the usual action of the system and S_m is the action modified by the addition of a monopole.

2. RESULTS

Instead of $\langle\mu\rangle$ it is numerically convenient to measure:

$$\rho = \frac{\mathrm{d}}{\mathrm{d}\beta}\log\langle\mu\rangle = \langle S\rangle_S - \langle S_m\rangle_{S_m} \quad (3)$$

and since $\langle\mu\rangle = 1$ for $\beta = 0$

$$\langle\mu\rangle = \exp\int_0^\beta \rho(\beta')\mathrm{d}\beta' \ . \quad (4)$$

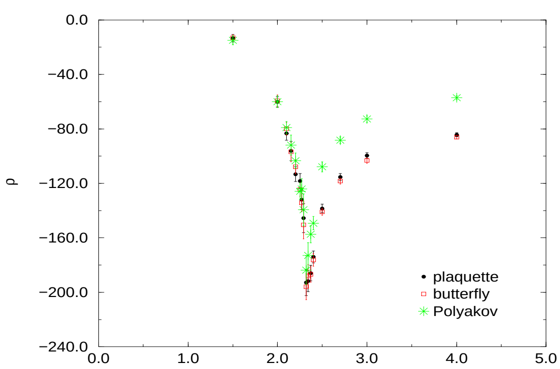

Figure 1. ρ vs. β for different abelian projections in $SU(2)$.

In fig. 1 we show the typical behavior of ρ vs. β for $SU(2)$ gauge theory and for different abelian projections. These data have been obtained on a Quadrics Machine.

Our main results are:

1) Different abelian projections show the same behavior.

2) Enlarging the spatial size of the lattice (at fixed temporal size) ρ remains finite for $\beta < \beta_c$, i.e. μ remains different from zero in the infinite volume limit. The asymptotic value at large β's diverges to $-\infty$ with the spatial dimension, so that μ goes to zero in the infinite volume limit in the deconfined phase $\beta > \beta_c$.

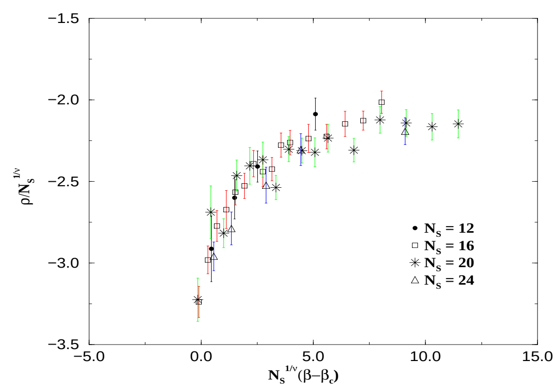

Figure 2. Finite size rescaling of data according eq. 5 with $\nu = 0.63$. Plaquette projection.

Finite size scaling requires that in the critical region $\beta \simeq \beta_c$

$$\rho/L^{1/\nu} = f(L^{1/\nu}(\beta-\beta_c)) \quad (5)$$

with ν the critical index related to the correlation length

$$\xi \sim (\beta_c - \beta)^{-\nu} \ . \quad (6)$$

How well scaling is obeyed is shown in fig. 2 where data are rescaled according to eq. 5 with $\nu = 0.63$ and $\beta_c = 2.295$ [15]. This is in agreement with the expectation that $SU(2)$ presents a phase transition of second order and belongs to the same universality class as 3d Ising model, where $\nu = 0.631(1)$.

3) The same analysis can be done in $SU(3)$. In this case there are two independent monopoles related to the two independent diagonal generators of SU(3) group for a given abelian projection. The corresponding ρ's are plotted for the Polyakov projection in fig. 3 and coincide within

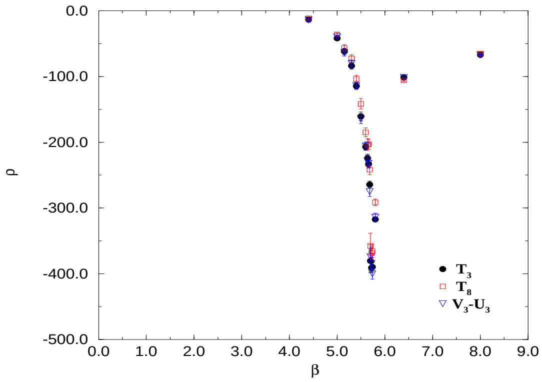

Figure 3. ρ for different monopoles of the same abelian projection in $SU(3)$. Polyakov projection.

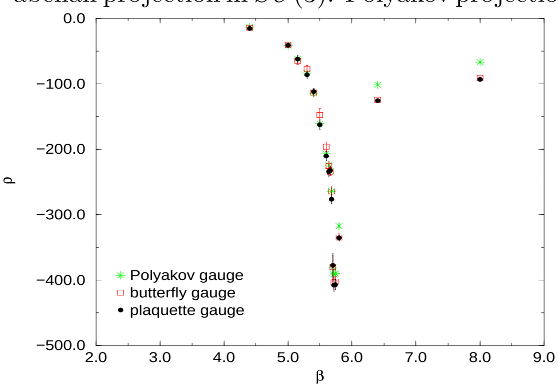

Figure 4. ρ for different abelian projections in $SU(3)$.

errors. Like for $SU(2)$, there is no difference between different abelian projections (Polyakov, spatial plaquette and butterfly) (see fig 4).

The finite size scaling analysis for $SU(3)$ is on the way. The expectation is $\nu = 1/3$ the phase transition being of first order.

3. CONCLUSIONS

Studying the symmetry properties of $SU(2)$ and $SU(3)$ gauge theories by means of a disorder parameter we find evidence that monopoles do condense in the confined phase.

Monopole condensation takes place in all the abelian projections we have studied. This shows that QCD vacuum is more complicated than a simple $U(1)$ superconductor.

In $SU(2)$ the scaling behavior of the disorder parameter gives a critical index equal to that of 3d Ising model.

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REFERENCES

1. G. 't Hooft, in "High Energy Physics", EPS International Conference, Palermo 1975, ed. A. Zichichi.
2. S. Mandelstam, *Phys. Rep.* **23C** (1976), 245.
3. M. Creutz, *Phys. Rev.* **D21** (1980), 2308.
4. R.W. Haymaker, J. Woziek, *Phys. Rev.* **D36** (1987), 3297.
5. A. Di Giacomo, G. Paffuti, *Phys. Rev.* **D56** (1997), 6816.
6. G. 't Hooft, *Nucl. Phys.* **B190** (1981), 455.
7. G. 't Hooft, *Nucl. Phys.* **B79** (1974), 276.
8. A.M. Polyakov, *JEPT Lett.* **20** (1974), 894.
9. L.P. Kadanoff, H.Ceva, *Phys. Rev.* **B3** (1971), 3918.
10. E. Marino, B. Schroer, J.A. Swieca, *Nucl. Phys.* **B200** (1982), 473.
11. J. Fröhlich, P.A. Marchetti, *Commun. Math. Phys.* **112** (1987), 343.
12. A. Di Giacomo, D. Martelli and G. Paffuti, these proceedings.
13. A. Di Giacomo hep-lat/9802008
14. A.Di Giacomo, B. Lucini, L. Montesi and G.Paffuti, in preparation.
15. J. Fingberg, U. M. Heller and F. Karsch, *Nucl. Phys.* **B392** (1993), 493.